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COMPARISON OF THE EFFECTS BETWEEN SINGLE AND MULTIPLE-SCATTERING ON CODA WAVES FOR LOCAL EARTHQUAKES

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Abstract:

The effects of multiple-scattering on coda waves have been investigated by an extension. of the single-scattering theory. The contributions to the coda power due to multiple-scattering from a uniform distribution of isotropic scatterers in a two-dimensional infinite elastic medium have been numerically evaluated. The results show that at shorter lapse time, the coda power is well explained by the single-scattering theory. At longer lapse time, this is inadequate and the effects of multiple-scattering need to be considered. The neglect of multiple-scattering gives rise to an overestimate on of the values of the quality factor (Q) bya factor of 3.85.

Introduction:

Knopoff and Hudson (1964) treated the problem of forward scattering of P- and S-waves by a randomly inhomogeneous elastic medium and showed that the scattered waves are primarily of the S-type. Aki (1969) considered the back scattering of the same wave-types by heterogeneities in the lithosphere. He showed that the coda waves observed for local earthquakes can be explained in terms of the back scattering of the primary waves in the lithosphere.

Using coda waves for local earthquakes, Aki and Chouet (1975), Rautian arid Khalturin (1978), Aki (1980a,b), Herrmann (1980) and Singh (1981) investigated the quality factor (Q) of seismic waves for frequencies greater than 0.1 Hz. Aki (1981a) synthesized the results of Q measurements and showed that the Q values of coda waves closely agree to those measured for S-waves. Also, he showed that the seismograms for coda and S-waves exhibit identical station-site effects. Integrating these observations with the results on the mechanism for the generation of coda waves mentioned above, Aki (1981b) concluded that the coda waves are S-to-S back-scattered waves.

The studies cited here, including those of Sato (1977a,b; 1978) and Kopnichev (1975, 1977) are based on the single-scattering theory. This theory is valid provided the mean free paths of the waves between the scatterers are greater-than the travel distances from the scatterers to receiver. However, there may be physical situations where the above conditions are not valid. For such cases, contribution from multiple-scattering to the seismic coda may become important.

In the present study, we therefore considered the effects of multiple-scattering on coda waves for a two-dimensional geometry. We approached the problem by following the single-scattering theory of Aki (1969) and Aki and Chouet (1975). "However, to maintain a continuity of the presentation with the works of Aki and colleagues, we first restate the problem of single-scattering and then proceed to compute the effects of higher-order scattering.

Mathematical Preliminaries: Single-Scattering

Consider a two-dimensional infinite elastic medium in which numerous, statistically isotropic scatterers are distributed uniformly. For this physical condition, we represent the energy due to single-scattering per unit area in the form:

$$\frac{\sigma|\phi(\omega|r_n)|^2}{2\pi r_n} \tag{1}$$

where σ is the scattering cross-section of the scatterer at distance r_n

from the station, $\phi(\omega|r_n)$ is the amplitude spectrum of the incident wave at r_n. The power spectral density, then can be expressed as

$$p(\omega|t)\Delta t = \sum_{r<r_n<r+\Delta r} \frac{\sigma|\phi(\omega|r_n)|^2}{2\pi r_n} \tag{2}$$

We assumed in (2) that the scatterers with isotropic cross-section σ are distributed in the area bounded by r and $r+\Delta r$. Also, we assumed that both the incident and scattered waves are of the same kind and share the same propagation velocity (v) and the station (receiver) and source of the primary waves are located at the same place. The scattered waves from the scatterers in (r, $r+\Delta r$) arrive at the station in the time interval (t, $t+\Delta t$), where $t=2r/v$ and hence,

$$At = 2\Delta r/v \qquad \{3)$$

in (3), it is implicit that the source dimension of the primary waves is small in comparison to volt.

Let n_o be the density of scatterer per unit area in $(r,r+\Delta r)$. In terms of n_0, (2) becomes

$$p(\omega|t)\Delta t = \frac{2\pi r\Delta r n_o\sigma}{2\pi r}|\phi(\omega|r)|^2$$

$$\text{or} \quad p(\omega|t)\Delta t = n_o\sigma\Delta r|\phi(\omega|r)|^2 \tag{4}$$

Substituting (3) in (4), we get

$$p(\omega|t) = \frac{vn_o\sigma}{2}|\phi(\omega|r)|^2 \tag{5}$$

For the conservation of energy (E), we have

$$E \sim 2\pi r|\phi(\omega|r)|^2 = 2\pi r_o \, |\phi(\omega|r_o)|^2$$

or
$$|\phi(\omega|r)|^2 = |\phi(\omega|r_o)|^2 (\frac{r_o}{r}) \qquad (6)$$

where $\phi(\omega|r_o)$ is the amplitude spectrum at the reference distance r_0.

In terms of the quality factor Q, the fractional loss of energy due to scattering and intrinsic absorption per cycle is $2\pi/Q$ and the attenuation in power during the time period t is $\exp(-\omega t/Q)$, where ω is the angular frequency. Incorporating the attenuation factor and (6) in (5), we get

$$p(\omega|t) = \frac{n_o r_o \sigma |\phi(\omega|r_o)|^2}{t} \exp(-\omega t/Q). \qquad (7)$$

where we replaced v/2 by r/t.

Multiple-Scattering:

In order to explain the computational procedures used here, we first consider the effects of double and triple-scattering.

(a) Double-Scattering:

For this case we consider the contribution to the coda of the scatterers in the two-dimensional area $2r \leq r_1 + r_2 + r_3 \leq 2(r+\Delta r)$. Here $r = vt/2$, r_1 and r_2 are the distances between the source (receiver occupies the same place as the source) and the first scatterer and between the first and second scatterers, respectively. r_3 is the distance between the second scatterer and the receiver (Fig.1).

We consider next, an uniform distribution of the second scatterer, thereby constituting a set of scatterers in the area between two concentric ellipses (Fig. 1) with foci at S and A. The first scatterer is held fixed for the time being at r_1. The major axes of the inner and outer ellipses" are then, $2r-r_1$ and $2(r+\Delta r)-r_1$, respectively. For the scatterers between the two ellipses, there are two sources of incident waves--one at S represented by the source of the primary waves and the other at A due to scattering of the primary waves by the first scatterer.

The scattered waves originating at A can be expressed in the form:

$$\sigma|\phi(\omega|r_1)|^2 \tag{8}$$

where $\phi(\omega|r_1)$ is the amplitude spectrum of the primary waves at r_1 and σ is the cross-section of the scatterer situated at this point.

In terms of a reference distance r_o, (8) becomes

$$\sigma|\phi(\omega|r_o)|^2(\frac{r_o}{r_1}) \tag{9}$$

Using (9) and following the same approach as used for the single-scattering case, we can represent the square of the amplitude spectrum at r for the double-scattering case in the form

$$|\phi(\omega|r)|^2 = \frac{\sigma^2}{2\pi r_2 2\pi r_3} (\frac{r_o}{r_1})|\phi(\omega|r_o)|^2 \exp(-\omega t/Q) \tag{10}$$

(10) represents the contribution of a pair of scatterers, one being in the area between the two ellipses and the other at r_1 from the receiver.

Consider an elemental area AA (Fig. 1) with n_o as the density of scatterer. The elemental area AA can be expressed as

$$\Delta A = r_3 \Delta\theta \Delta r_3 \tag{11}$$

The equation of the inner ellipse in polar coordinates (r,θ) is

$$r_3 = \frac{(a^2-c^2)}{a-c\ \cos\theta} \qquad (12)$$

where θ is the angle between r_1 and r_3; it is measured counterclockwise. a is the semi-major axis and is given by

$$a = \frac{2r-r_1}{2} \qquad (13)$$

or

$$ha = \Delta r \qquad (14)$$

For fixed" $c = \frac{r_1}{2}$, we have

$$\Delta r_3 = \frac{\partial r_3}{\partial a} \Delta a \qquad (15)$$

For fixed e, using (12) we write (15) in the form

$$\Delta r_3 = (1 + \frac{c^2 \sin^2\theta}{(a-c\ \cos\theta)^2})\Delta a \qquad (16)$$

Substituting (14) and (?6) in (11), we get

$$\Delta A = \left[1 + \frac{c^2\ \sin^2\theta}{(a-c\ \cos\theta)^2}\right] r_3 \Delta r \Delta\theta \qquad (17)$$

Thus, the contribution of all scatterers in the area between the two ellipses can be expressed in the form

$$n_o \int_A |\phi(\omega|r)|^2 dA \xrightarrow{} \frac{}{8\pi^2 r_1} n_o \sigma^2 |\phi(\omega|r_o)|^2 \exp(-\omega t/Q)$$

$$\int_0^{2\pi} \left[1 + \frac{c^2 \sin^2\theta}{(a-c\cos\theta)^2}\right] d\theta \tag{18}$$

where we replaced Δr by $v\Delta t/2$. A is the area between the two ellipses.

The number of first scatterers in the interval $(r_1, r_1+\Delta r_1)$ is $2\pi r_1 \Delta r_1 n_o$. The power spectral density of the waves arriving in the time interval $(t, t+\Delta t)$ for double-scattering then becomes

$$p(\omega|t)\Delta t = \int_0^{r} 2\pi r_1 n_o dr_1 n_o \int_A |\phi(\omega|r)|^2 dA \tag{19}$$

Using (18) in (19), we get

$$p(\omega|t) = \frac{1}{4\pi} n_o^2 \sigma^2 r_o v |\phi(\omega|r_o)|^2 \exp(-\omega t/Q) K_2 \tag{20}$$

where

$$K_2 = \int_0^{r} dr_1 \int_0^{2\pi} \frac{1}{(2r-r_1-r_3)} \left[1 + \frac{r_1^2 \sin^2\theta}{(2r-r_1-r_1\cos\theta)^2}\right] d\theta \tag{21}$$

Substituting $t = \frac{r_1}{r}$ in (21), we get

$$K_2 = \int_0^{1} dt \int_0^{2\pi} \frac{2-t(1+\cos\theta)}{2-(2t-t^2)(1+\cos\theta)} \left[1 + \frac{t^2 \sin^2\theta}{\{2-t(1+\cos\theta)\}^2}\right] d\theta \tag{22}$$

b. Triple-Scattering.:

In this case, the scatterers being located at r_1, r_2 and r_3 satisfy the condition $2r \leq r_1+r_2+r_3+r_4 \leq 2(r+\Delta r)$, where $r = vt/2$, r_1, r_2 and r_3 are the distances between the receiver and first scatterer, between first and second scatterer and between second and third scatterer, respectively. r_4 is the distance between the third scatterer and the receiver. As before, the square of the amplitude spectrum at r can be expressed in the form

$$|\phi(\omega|r)|^2 = \frac{\sigma^3}{2\pi r_2 2\pi r_3 2\pi r_4} \left(\frac{r_o}{r_1}\right) |\phi(\omega|r_o)|^2 \exp(-\omega t/Q) \tag{23}$$

where the quantities on the left- and right-hand sides denote the" same quantities as in (10).

For computational purposes, we fix the positions of the first and . second scatterers at r_1 and r_2, respectively, and take into account the third scatterers distributed in the area between two concentric ellipses, such that $(2r-r_1-r_2) < r_3+r_4 < [2(r+\Delta r)-r_1-r_2]$ is satisfied. Next, following a similar approach as used for the double-scattering case, we integrate with respect to r_2 and r_1 for $0 \leq r_2 \leq (r-r_1)$ and $0 \leq r_1 \leq r$, respectively. The power spectral density, thus can be expressed in the form

$$p(\omega|t)\Delta t n_o r_1 dr_1 \int_0^{r} \int_0^{r-r_1} 2\pi n_o r_2 dr_2 \int_A \frac{\sigma^3 n_o}{8\pi^3 r_2 r_3 r_4} \left(\frac{r_o}{r_1}\right) |\phi(\omega|r_o)|^2 \exp(-\omega t/Q)\,dA \tag{24}$$

or

$$p(\omega|t) = \frac{n_o^3 \sigma^3 r_o v |\phi(\omega|r_o)|^2 \exp(-\omega t/Q)}{4\pi} \int_0^{r} dr_1 \int_0^{r-r_1} dr_2 \int_0^{2\pi} \frac{1}{(2r-r_1-r_2-r_4)} \frac{\partial r_4}{\partial a} d\theta \tag{25}$$

'here 'A $\frac{\partial r_4}{\partial r}$ $r_4$dadθ, da = 'r and

$$r_4 = a - \frac{a^2-c^2}{a-c\cos\theta} \qquad (26)$$

$$2a = 2r-r_1-r_2 \qquad (27)$$

$$2c = r_1+r_2 \qquad (28)$$

Using (26'), (27) and (28) in (25), we get

$$p(\omega|t) = \frac{1}{4\pi} n_o^3\sigma^3 r_o rv|\phi(\omega|r_o)|^2 \exp(-\omega t/Q)K_3 \qquad (29)$$

where

$$K_3 = \frac{1}{r}\int_0^r dr_1 \int_0^{r-r_1} dr_2 \int_0^{2\pi} \frac{1}{(2r-r_1-r_2-r_4)} [1 + \frac{c^2\sin^2\theta}{(a-c\cos\theta)^2}]d\theta \qquad (30)$$

or

$$K_3 = \int_0^1 dt_1 \int_0^{1-t_1} dt_2 \int_0^{2\pi} \frac{2-(t_1+t_2)(1+\cos\theta)}{2-\{2(t_1+t_2)-(t_1+t_2)^2\}(1+\cos\theta)}$$

$$\left[1 + \frac{(t_1+t_2)^2\sin^2\theta}{\{2-(t_1+t_2)(1+\cos\theta)\}^2}\right]d\theta \qquad (31)$$

'here $t_1 = \frac{r_1}{r}$ and '2 'F-

In general, the contribution to the seismic coda for kth order scattering can be expressed in the form

$$p_k(\omega|t) = \frac{1}{4\pi} r_o r^{(k-2)} n_0^k\sigma^k v|\phi(\omega|r_0)|^2 \exp(-\omega t/Q)K_k \qquad (32)$$

where

$$K_k = \frac{1}{r^{k-2}} \int_0^{r} dr_1 \int_0^{r-r_1} dr_2 \cdots \int_0^{r-r_1 \cdots - r_{k-2}} dr_{k-1} \int_0^{2\pi} \frac{1}{(2r-r_1 \cdots - r_{k-1} - r_{k+1})} \left[1 + \frac{c^2 \sin^2\theta}{(a-c\cos\theta)^2}\right] d\theta \qquad (33)$$

or

$$K_k = \int_0^{1} dt_1 \int_0^{1-t_1} dt_2 \cdots \int_0^{1-\tau_1} dt_{k-1} \int_0^{2\pi} \frac{2-\tau_2(1+\cos\theta)}{2-\left\{2\tau_2-\tau_2^2\right\}(1+\cos\theta)} \left[1 + \frac{\tau_2^2 \sin^2\theta}{\left\{2-\tau_2(1+\cos\theta)\right\}^2}\right] \qquad (34)$$

where $t_i = \frac{r_i}{r}$, $\tau_1 = \sum_{i=1}^{k-2} t_i$, $\tau_2 = \sum_{i=1}^{k-1} t_i$.

Instead of solving the integrals for K_k (k= 1, 2, . ..) analytically, we evaluated them numerically. The values of K_k for different k up to k = 7 are given in Table 1, which were obtained by Gauss's integration method. Since the value of $p_k(\omega|t)$ decreases rapidly with increasing order of scattering, we restricted the computation to $k \le 7$.

The power spectral density of the coda waves due to multiple scattering can thus be expressed in the form .

$$P(\omega|t) = P_s(\omega|t) + P_m(\omega|t) \qquad (35.)$$

In (35), $P_s(\omega|t)$ and $P_m(\omega|t)$ represent the contributions for single and multiple scattering, respectively, where

$$P_s(\omega|t) = \frac{1}{t} n_o \sigma r_o |\phi(\omega|r_o)|^2 \exp(-\omega t/Q) \tag{36}$$

and $P_m(\omega|t)$ for nth order scattering is

$$P_m(\omega|t) = n_o^2 \sigma^2 v r_o |\phi(\omega|r_o)|^2 \exp(-\omega t/Q) \sum \left[\alpha_k \beta^{k-2}\right] \tag{37}$$

where α_k are numerical 'constants which are given in Table 1 for different k and $\beta = n_o \sigma r$.

It may be noted that for the two-dimensional geometry, σ and n_0 have dimensions of length L and L^{-2}, respectively, for unit mass of each scatterer. Thus $\beta = n_o \sigma r$ is a dimensionless quantity. Using the values of α_k (k = 2, 3,".. .) from Table 1, the polynomial in the square bracket in (37) can be fitted by the function $\exp(0.74\beta)$ in the interval $0 \leq \beta \leq 3$ for 99 percent confidence limits. We thus rewrite (37) in the form

$$P_m(\omega|t) = (n_o\sigma)^2 r_o v |\phi(\omega|r_o)|^2 \exp(0.74\beta - \omega t/Q)$$

Or

$$P_m(\omega|t) = S \exp(0.74\beta - \omega t/Q), 0 \leq \beta \leq 3 \tag{38}$$

where

$$S = (n_o\sigma)^2 r_o v |\phi(\omega|r_o)|^2 \tag{39}$$

Using (39), we rewrite (36) in the form

$$P_s(\omega|t) = \frac{S}{2\beta} \exp(-tit/Q) \tag{40}$$

Substituting (38) and (40) *in* (35), we get

$$P(\omega|t) = S \exp(-\omega t/Q)\left[\frac{1}{2\beta} + \exp(0.74\beta)\right] \tag{41}$$

Since the dimensionless quantity β represents the loss in power due to scattering, we express the quality factor Q_s due to single scattering in the form

$$Q_s = \frac{\omega t}{n_o \sigma r}$$

or

$$Q_s = \frac{\omega t}{\beta} \tag{42}$$

where ω is the angular frequency. Recall that the quality factor Q represents the combined loss due to single scattering and intrinsic absorption. Let Q_i represent the quality factor for intrinsic absorption. In terms of Q_s and Q_i, Q becomes

$$\frac{1}{Q} = \frac{1}{Q_s} + \frac{1}{Q_i} \tag{43}$$

Substituting (42) and (43) in (38) and (40), we get

$$P_m(\omega|t) = S \exp\left[-\omega t\left(\frac{0.26}{Q_s} + \frac{1}{Q_i}\right)\right] \tag{44}$$

and

$$P_s(\omega|t) = (2\omega t)^{-1} Q_s S \exp\left[-\omega t\left(\frac{1}{Q_s} + \frac{1}{Q_i}\right)\right] \tag{45}$$

From the time dependence t^{-1} of (45), it follows that at short lapse time t (time measured from the origin time of the source) and for a given value of ω, the contribution from single-scattering dominates the power spectral density of coda waves. However, as t increases, the contribution to the power spectral density from multiple-scattering becomes dominant. Thus, at sufficiently large values of t, the determination of Q values on the basis of single-scattering theory yields an overestimation of Q values by a factor of 3.85 (reciprocal. of 0.26), as can be seen from (44).

In order to elaborate the above observations, we computed $P(\omega|t)/S$, $P_m(\omega|t)/S$ and $P_s(\omega|t)/S$ as a function of β and $\bar{Q}$ = 1.0, 0.5 and 0.1 from (41), (44) and (45). Here $\bar{Q} = Q/Q_s$, where Q and Q_s represent the same quantities as in (43). The results of computations are given in Figs. 2a, band c.

It may be noted in Figs. 2a, b and c that the graphs of $P_s(\omega|t)/S$ and $P_m(\omega|t)/S$ as a function of β for all the three values of $\bar{Q}$ intersect for a value of $\beta = \beta_c$, where $\beta_c = 0.38$ corresponding to a time $t = t_c$. For $\beta > 0.38$ and $\bar{Q}$ of 1.0 and 0.5, the contribution to the coda from multiple-scattering starts dominating with increasing values of β. However, single-scattering accounts well the-power in the coda for $\beta = n_o\sigma r < \beta_c$ or for travel time $t_c < 0.76(n_o\sigma v)^{-1}$.

The quantity $n_o\sigma$ has been called turbidity by Aki and Chouet {1975); it has the dimension of the reciprocal of mean-free path. For the LASA area in Montana, Aki (1980) obtained a value of 0.008 km^{-1} for $n_o\sigma$. This value with v = 3.5 km/see (velocity of S-waves in the upper part of the lithosphere} and $\beta_c = 0.38$ yields a travel time (t_c) of28 sec. Thus, for travel time (t) in the range of 100 to 200 see, at least for the LASA area, the contribution of multiple-scattering to the coda section of the seismograms for local earthquakes would be important.

For $\overline{Q} = 0.1$ (Fig. 2c), the power in coda decreases rapidly to an insignificant level at $\beta = \beta_c$ compared to the other two values of $\overline{Q}$. Therefore, the overall contribution of single scattering to the coda is more important than those of multiple scattering; thus for this value of $\overline{Q}$, the effects of multiple scattering can be neglected.

Conclusions:

In this study, the single scattering theory of Aki (1969) for coda - waves has been extended to the multiple-scattering case for a two-dimensional elastic medium consisting of a uniform distribution of numerous but statistically isotropic scatterers. For this physical condition of the medium, the power spectral density of coda waves for the nth order scattering has been expressed in integral form. Numerically integrating this expression up to 7th order scattering the contributions to the coda power from single- and multiple-scattering are compared.

The results of the comparison show that at shorter lapse time $t < t_c$, the properties of coda waves are well explained by the single-scattering theory. However, for lapse time $t > t_c$, the single-scattering theory is not adequate in explaining the coda waves. Rather, the contributions to the coda power from multiple-scattering become important. At longer lapse time $t \gg t_c$, the determination of the quality factor Q on the basis of single-scattering theory yields an overestimation of the Q values by a factor of 3.85.

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TABLE 1.

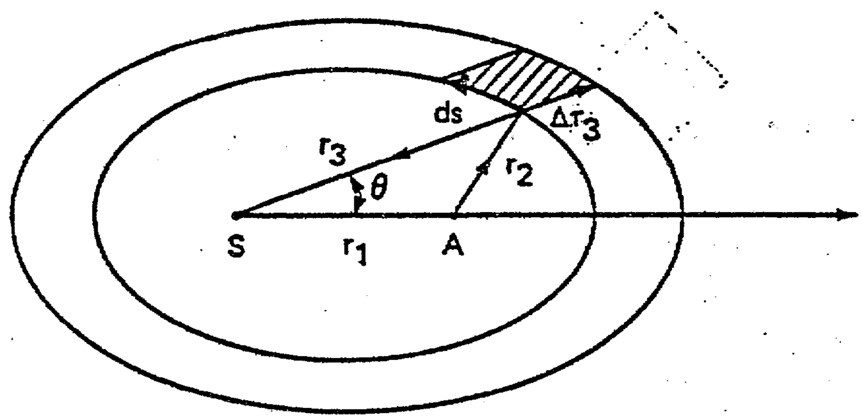
Values of K_k and α_k are shown for different values of k.

k	K_k	α_k
2	12.5	1.0
3	8.31	0.66
4	3.26	0.26
5	0.921	0.074
6	0.203	0.016
7	0.035	0.0028

Figure Captions:

Figure 1. The geometry for the distribution of scatterers.

Figure 2. "The relationship of coda power as a function of β and $\bar{Q}$ are shown for single- and multiple-scattering. The values of $P(\omega|t)/S$, $P_s(\omega|t)/S$ and $P_m(\omega|t)/S$ are shown as a function of β. (a) $\bar{Q} = 1.0$. (b) $\bar{Q} = 0.5$, and (c) $\bar{Q} = 0.1$. Note that the total power spectral density of coda waves is $P(\omega|t) = P_s(\omega|t) + P_m(\omega|t)$, where $P_s(\omega|t)$ and $P_m(\omega|t)$ are the power spectral densities due to single- and multiple-scattering, respectively.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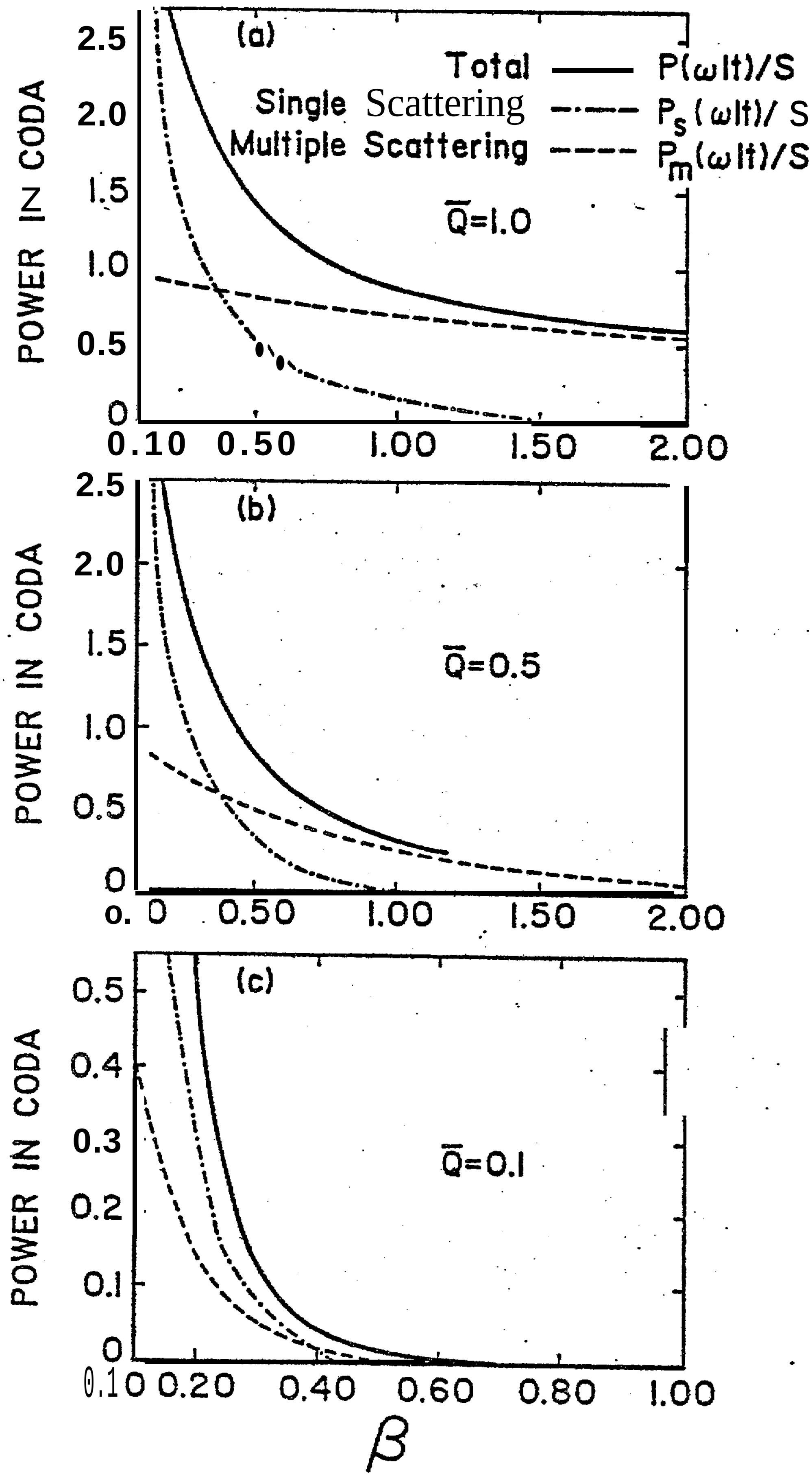
(a)
Total P(ω|t)/S
Single Scattering P_s(ω|t)/S
Multiple Scattering P_m(ω|t)/S
Q̄=1.0
(b)
Q̄=0.5
(c)
Q̄=0.1
POWER IN CODA
β