Instruction Manual

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1 Introduction

Submesoscale (SMS) features of the order of 1-10 km (the higher range of submesoscales) can impact substantially the tracer dispersion, due in part to the frontal instabilities they trigger at the edges of mesoscale features. Recent observations and high resolution simulations of the Gulf of Mexico (GoM) have confirmed their existence, in particular in the GoM interior where mesoscale features are ubiquitous. They tend to emerge during winter and autumn when the mixed-layer depth has reached a critical depth.

The purpose of the Lagrangian stochastic model (hereafter LSM) developed here is to enhance the relative dispersion statistics of surface particle trajectory ensembles, since these metrics are closely related to tracer dispersion and mixing. More specifically, its implementation aims at approaching the scale-dependent Finite Scale Lyapunov Exponent of a very high resolution simulation by enhancing its value at the submesoscales while preserving it at the mesoscales. The LSM is based, however, on single particle statistics, and impacts only indirectly the relative dispersion.

The Lagrangian parameterization is to be applied to a mesoscale eddy-resolving coastal ocean model where the grid resolution is too coarse to resolve submesoscale motions. The LSM developed in this code attempts to correct the Lagrangian transport of the ocean model by mimicking the effect those SMS features would have on the relative dispersion from a statistical standpoint.

The coastal model considered here is the HYCOM $1/25^o$ free run configured for the Gulf of Mexico. The daily outputs of the surface horizontal velocities are used for particle advection. The LSM-3 Lagrangian stochastic model is implemented for the GoM interior, and requires an intrinsic flow decomposition from the ocean model prior to running the code, including the resulting Markov-1 statistical parameters $(\sigma_m^u, \sigma_m^v, \tau_m^u, \tau_m^v)$ of the ocean model. For the continental shelves, the smaller kinetic energy and relative dispersion favor all three LSMs including the LSM-1 (Random Walk), the LSM-2 (Random Flight) and the LSM-3, albeit with a weak turbulent component. The transition from the interior to the shelf and vice-versa is done at a critical depth around 100 m, where all LSM parameters are set to zero.

The manual is organized as follows: the LSM subroutines are described in Section 2. The codes to run prior to the parameterization are described in Section 3 and the LSM code is presented in Section 4. How the LSM has affected the Markov-1 parameters and relative dispersion is then part of the post-processing described in Section 5. Section 6 contains the sequence of instructions to run all the codes. The secondary subroutines are listed alphabetically in Section 7, while the main variables and parameters are described in Section 8.

2 LSM subroutine descriptions

2.1 Random walk for two-particles (LSM-1)

There is a choice of two slightly different correlated random-walks:

(A): **subroutine RwLSM1a(xa,ya,dx,dy,kcount,ip0,xprime1,yprime1,x01,y01)**

If $Ldecor \ll \mathrm{O}(Lkick)$: then the random walk is spatially uncorrelated.

In that instance, the subroutine adds a random kick **dx** at every integration time step Dt as follows:

$$\mathbf{dx} = Lkick\ \mathbf{dW}\ , \tag{1}$$

where the random kick is characterized by an amplitude $Lkick$ and a normal distribution dW of zero-mean and standard deviation of unity.

Here the random number is read from the ASCII file **randomNormal-1.txt** opened in the main code.

$Lkick$ and Dt are specified in the parameter file in units of km and sec respectively.

In terms of magnitude, the random walk generates velocity fluctuations with a standard deviation $\sigma = Lkick/Dt$. For example, if $Lkick = 1$ km and $Dt = 2$ hrs, $\sigma \approx 14$ cm/s, about an order of magnitude smaller than the Loop current velocities. The corresponding diffusivity $K = Lkick^2/(2Dt)$.

If $Ldecor \sim \mathrm{O}(\mathrm{km})$, then the subroutine constrains the random component of the second particle of each pair by the spatial correlation term:

$$AA = exp\left(-\frac{drel^2}{2\ Ldecor^2}\right), \tag{2}$$

where $Ldecor$ is the decorrelation space scale specified in the parameter file, and $drel$ is the relative distance of particle-pairs.

The random kick of the second particle is therefore:

$$\mathbf{dx}_2 = AA\ \mathbf{dx}_1 + (1 - AA)\ Lkick\ \mathbf{dW}, \tag{3}$$

dx_1 being the random kick of the first particle.

(B): **subroutine RwLSM1b(xc,yc,dx,dy,kcount,ip0,xpr,ypr)**

This subroutine takes onto account all nearby particles and for all particles.

if N particles are within the decorrelation length-scale of a given particle, then its random kick dx can be expressed as follows:

$$\mathbf{dx} = \frac{1}{N}\sum_{i=1}^{N} AA_i \mathbf{dx}_i + \left(1 - \frac{1}{N}\sum_{i=1}^{N} AA_i\right)\ Lkick\ \mathbf{dW}. \tag{4}$$

2.2 Random Flight (LSM-2)

The LSM2 for two particles is implemented in:

subroutine EulerLSM2(xa,ya,kcount,ip0,dWx1,dWy1,upa,vpa,u1,v1,u2,v2, it,iti,dx,dy)

This Lagrangian stochastic model is Markovian in both the distance and velocity. It adds a random turbulent velocity $\mathbf{v}'$ modulated in time by a decorrelation time scale τ. The turbulent velocity increment $\mathbf{dv}'$ along a trajectory is based on the formulation (here in the zonal direction):

$$du' = -u'\frac{Dt}{\tau} + \sigma\sqrt{\frac{2Dt}{\tau}}dW \tag{5}$$

The standard deviation of the turbulent velocities *(sigu, sigv)* and decorrelation time-scales *(tauu, tauv)* are specified in the parameter file.

The correspondence with LSM-1 in terms of diffusivity ($K = \sigma^2\ \tau$) implies that for a given decorrelation time scale τ, σ should be chosen such that

$$\sigma \sim \frac{Lkick}{Dt}.\sqrt{\frac{Dt}{2\tau}}, \tag{6}$$

since the decorrelation time-scale of a random walk is Dt/2.

The spatial correlation is also implemented for the second particle as follows:

$$du_2' = -u'\frac{Dt}{\tau} + (1 - AA)\sqrt{\frac{2Dt}{\tau}}dW_1 + AA\sqrt{\frac{2Dt}{\tau}}dW, \tag{7}$$

Where dW_1 is the random component of the first particle.

2.3 LSM-3

In the Lagrangian framework:

$$\frac{d\mathbf{X}_m}{dt} = \mathbf{U}_m(\mathbf{X}_m, t) + \mathbf{u}'_m(\mathbf{X}_m, t), \tag{8}$$

where $\mathbf{U}_m$ is the filtered model velocity corresponding to the slowly evolving mesoscale field, and $\mathbf{u}'_m$ is the remaining turbulent velocity field.

The corrected trajectories are obtained from the LSM-3 via the addition of the missing turbulent velocity component $\boldsymbol{\eta}$:

$$\frac{d\mathbf{X}_c}{dt} = \mathbf{U}_m(\mathbf{X}_c, t) + \mathbf{u}'_m(\mathbf{X}_c, t) + \boldsymbol{\eta}(t) \tag{9}$$

The missing component $\boldsymbol{\eta}$ modifies the Markov-1 parameters of the model, i.e.: the velocity fluctuation standard deviations and Lagrangian correlation times (σ_m, τ_m), to targeted/realistic parameters (σ_r, τ_r) that are assumed known.

Its evolution in time along each trajectory is governed by the following expression (in the zonal direction):

$$\frac{d\eta(t)}{dt} = a\frac{du'_m(t, \mathbf{r}_c(t))}{dt} + bu'_m(t, \mathbf{r}_c(t)) + c\eta(t)\,, \tag{10}$$

where

$$a = \frac{\sigma_r\sqrt{\tau_m}}{\sigma_m\sqrt{\tau_r}} - 1, \quad b = \frac{\sigma_r}{\sigma_m\sqrt{\tau_r\tau_m}} - \frac{1}{\tau_r}, \quad c = -\frac{1}{\tau_r}\,, \tag{11}$$

The variance of the missing component η is given by

$$\overline{\eta^2} = \frac{(\sigma_r\sqrt{\tau_m} - \sigma_m\sqrt{\tau_r})^2 + (\sigma_r\sqrt{\tau_r} - \sigma_m\sqrt{\tau_m})^2}{\tau_r + \tau_m}\,, \tag{12}$$

which is zero if the model and the real parameters coincide.

(σ_m, τ_m) for both zonal and meridional directions are specified in the parameter file after being computed from the autocovariance functions in **prior.f**.

On the other hand (σ_r, τ_r) must be specified in the parameter file. In the example here, they are set to be twice the model parameters. Note: σ_r should not exceed $3\sigma_m$.

The two main subroutines of the LSM-3 are:

subroutine paramIMC(su_m,sv_m,tu_m,tv_m,su_r,sv_r,tu_r,tv_r, ax,bx,cx,ay,by,cy,sex,sey) .
This subroutine computes the parameters *a*, *b*, *c* (*ax, bx, cx and ay, by, cy* in the zonal and meridional directions, respectively) according to Eq.11, and estimates the standard deviation of the $\boldsymbol{\eta}$ components (sex, sey).

subroutine EulerLSM3(n,m,xc,yc,u1,v1,u2,v2,IT,ITI,DT,dx,dy,ex,ey,uM,vM, ax,bx,cx,ay,by,cy).
This subroutine integrates Eq.10 using the Euler advection scheme in the following order:

1. um'(xc(t0),t0) = um(xc(t0),t0) - umean(xc(t0))
2. xc(t0+Dt) = xc(t0) + Dt . [um(xc(t0),t0) + eta(t0)]
3. um'(xc(t0+Dt),t0+Dt) = um(xc(t0+Dt),t0+Dt) - umean(xc(t0+Dt))
4. eta(t0+Dt) = eta(t0) + Dt . [a (um'(xc(t0+Dt),t0+Dt) - um'(xc(t0),t0)) / Dt) + b . um'(xc(t0),t0) + c . eta(t0)]

3 Prior to running the LSM-advection code

Before applying the LSMs, the particle ensemble has to be advected by the OGCM to compute the Markov-1 statistical parameters for the LSM-3. It requires a decomposition of the flow, which is obtained by low-pass filtering the model velocities.

Note that the GoM interior and continental shelf are treated separately, based on a condition on the depth.

Other statistical metrics are computed for references, which are the scale-dependent Finite Scale Lyapunov Exponent ($\lambda(\delta)$) and the time-dependent relative dispersion ($D^2(t)$), for the interior and the shelf separately.

3.1 AdvecMod.f

An ensemble of particles are launched in the region of interest and advected by the OGCM velocities (HYCOM1/25°). **advecMod.f** advects *npar* particles from their initial positions specified in *flLaunch*, using a fourth order Runge-Kutta scheme for the time-integration and a third-order polynomial method for the spatial interpolation.
The parameters are specified in **paramMod.h**.
The trajectories are stored in **trajMod.out**.

3.2 Prior.f

The parameters and variables are specified in **paramPrior.h**.

(1) If the LSM-3 is used on the continental shelf, the flow decomposition and Lagrangian statistical parameters must be computed twice with different parameters.
It is specified by the integer LSM (=3 if the LSM-3 is used on the shelf).

A critical depth (*depthC*) separates the GoM interior from the continental shelf and must be specified in the parameter file.

(2) The code generates low-passed velocity fields either from a Gaussian spatial average (subroutine **filterG**) or from a temporal moving average (subroutine **filterLPt**).

Both the choice and strength of the filter are specified in **paramPrior.h**.
filter choice: *FILTER* = 'TEMP' or 'GAUSS' for temporal or gaussian filter, respectively.
Filter strength:
np = number of days of the time window.
sigmaInt = Spatial Gaussian width in grid-points for the GoM interior.
sigmaShelf = Spatial Gaussian width for the shelf.

The filtered velocities are stored in a subdirectory of the directory **filTest/**. The name of each filtered velocity field is specified in the main code under *flvelLP*.

(3) The model Markov-1 parameters corresponding to the chosen filter are computed in subroutine **autocorr**. They are derived from the autocovariances ($c_{uu}(\tau),\ c_{vv}(\tau)$), with ($sigu_m,\ sigv_m$) corresponding to ($c_{uu}(0),\ c_{vv}(0)$), and ($tauu_m,\ tauv_m$) corresponding to $(c_{uu}(\tau),\ c_{vv}(\tau)) \sim 1/3(c_{uu}(0),\ c_{vv}(0))$.

Another set of parameters is computed for the shelf if the LSM-3 is also used. A different flow decomposition is used on the shelf, fixed to be the Gaussian filter with a strength of 1.5 grid-point. The name of the filtered velocity field is specified under *flvelLPs*.

In both cases, the file names are specified in the code under *flAutoc* for the autocorrelation functions and *flSig* for the Markov-1 parameters.

(4) The subroutine **liap** computes $\lambda(\delta)$ of the model trajectories for both the interior and the shelf and saves the curve in *flliap*.

(5) Similarly, the subroutine **relDisp** computes $D^2(t)$ and stores the results in *flrd*.

Below are the main subroutines being called in **prior.f**:

subroutine filterLPt(flvelLP,lo)
This subroutine low-passes the output velocities with a temporal moving average, with a time window of *np* days (*np* is specified in the parameter file).
The filtered velocities are stored daily in the file *flvelLP*. *no* is the character number of the directory or path.

subroutine filterG(dx,dy,depth,flGvel,lo,sigma)
Low-passes the output velocities with a spatial Gaussian filter of width *sigma* (units in grid-points).
The filtered velocities are stored daily in the file *flGvel*. *no* is the character number of the directory.

subroutine autoCorr(dx,dy,xx,yy,ktm,firstDay,intv,flFilt,flAutoc,flSig)
Computes the autocorrelation functions of the detrended Lagrangian velocities. They are obtained from the trajectories grid-coordinates *(xx,yy)* and flow-decomposition which is function of the filtered velocities specified in *flFilt*.
The autocorrelation functions ($covuu(\tau)$, $covvv(\tau)$) are stored in *flAutoc* and the statistical parameters (σ_u, τ_u, σ_v, τ_v) are stored in *flSig*.

subroutine liap(xxl,yyl,ktm,depth,intv,flliap)
Computes the scale-dependent Finite Scale Lyapunov Exponent $\lambda(\delta)$ of the trajectories *(xxl,yyl)* with units in (lon,lat).
$\lambda(\delta)$ is stored in the file *flliap*.

subroutine relDisp(xxl,yyl,ktm,intv,flrd)
Computes the relative dispersion $D^2(t)$ of the trajectories *(xxl,yyl)* and stores the result in *flrd*.

4 LSM-advection code

The main code (**advecLSM.f**) advects an ensemble of particles, once or for repetitive launches, using two LSMs: one for the interior and one for the shelf.

It requires a priori one or two flow-decomposition and corresponding statistical parameters, which are computed by first running **advecMod.f** and **prior.f**.

4.1 Variables and parameters

The parameter values and declarations of variables are all specified in the file **paramLSM.h**. Their correspondences are explained in section 8.

4.2 AdvecLSM.f

The code advects an ensemble of *npar* particles with the model velocities combined with an LSM. The launch coordinates are read from the file *flIni*.

The transition from the interior to the shelf (and vice-versa) is done at a given critical depth (*depthC*), where the parameters and vectors of an LSM are always set to zero.

There are four main loops:

(1) Launch loop (ktkt).

- The launch can be reiterated (loop ktkt) with a different date and/or different initial $\boldsymbol{\eta}$s. Note that only 1 $\boldsymbol{\eta}_0$-vector is picked for all particles. For a total number *ktm0* of launches, each $\boldsymbol{\eta}_0$ vector is chosen from the following expression:

$$\boldsymbol{\eta}_0(ktkt) = \begin{pmatrix} \sigma_x^\eta cos\,(2\pi ktkt/ne) \\ \sigma_y^\eta cos\,(2\pi(ktkt+\phi)/ne) \;\; , \end{pmatrix} \tag{13}$$

where $(\sigma_x^\eta, \sigma_y^\eta)$ are the standard deviations of $\boldsymbol{\eta}$ in both horizontal directions, *ne* is the number of divisions on the $(\sigma_x^\eta, \sigma_y^\eta)$ ellipse, and ϕ is a phase.

(2) Model output loop (l).

- If only $\boldsymbol{\eta}_0$ varies with ktkt, the loop starts in the same day: *l0 = day0*. Otherwise for a launch reiterated every month: *l0 = day0+30*(ktkt-1)*.

- The code reads two consecutive model velocity fields for time-interpolation and the advection schemes.

- The low-passed velocities are also read for the LSM-3 of the GoM-interior and the continental shelf if needed.

(3) Model time-interpolation loop (it).

- The model velocities are linearly interpolated here to 2 hour-time increments corresponding to the advection time-step *Dt*. *iti* id the number of interpolations from two consecutive outputs.

- The same is done to the low-passed velocity fields.

(4) Particle-loop (ipp).

- The particles of indice *ipp* are advected with model interpolated velocities and LSMs.

- For the GoM interior, the particles are advected by calling the subroutine **EulerLSM3**. It corresponds to the LSM-3 combined with the Euler advection scheme for the Lagrangian equation and 3rd order polynomial routine for the spatial interpolation.

For the continental shelf:

- If LSM-1 is used on the shelf: The model advection and LSM-1 are implemented separately. A fourth-order Runge-Kutta scheme and 3rd order polynomial interpolation are used in the subroutine **RK4**, followed by either the **RwLSM1a** or **RwLSM1b** subroutine.

- If LSM-2 is used on the shelf: Both model advection and LSM-2 are implemented in the subroutine **EulerLSM2**. The Euler scheme is also used to solve the equation.

- If LSM-3 is on the shelf: here again, the subroutine **EulerLSM3** is called with the statistical parameters of the shelf.

5 Diagnostics

After the main LSM code has computed the parameterized trajectories, the performance of the LSMs is tested by making diagnostics and comparing with the model trajectories.

5.1 Post.f

The parameters and variables are specified in the file **paramPost.h**.

(1) The code reads one set of parameterized trajectories (here **traj.out.001**) and computes the autocorrelation functions and statistical parameters of the corrected trajectories on the GoM interior and continental shelf (if LSM=3).

The file names are specifed in the code under *flvelF* for the low-passed velocities, *flAutoc* for the autoccorrelation functions and *flSig* for the statistical parameters.

Upper and lower bounds (*dpmin*, *dpmax*) are given for the specification of the GoM interior and shelf.

A comparison with the model parameters is given by computing the ratio of *corrected* to *model* parameters. Most of the time, the ratio is different than the *target/model* ratio expected for homogeneous turbulence.

(2) Both scale-dependent FSLE and relative dispersion of the corrected trajectories are computed independently for the GoM interior and continental shelf:

The file names are also specified under *flLiap* for $\lambda(\delta)$ and *flrd* for $D^2(t)$.

6 Instruction Sequence

6.1 Model trajectories

to generate model trajectories:

In **paramMod.h**, specify:
- The day of launch (*day0*) and duration of advection (*lmt*).
- The number of particles to be launched (*npar*).

In **advecMod**, specify:
- The name of the launch file (*flLaunch*).
- Note that the particles must be launched by pairs or triplets for the relative dispersion metrics.

Compile and run **advecMod.f**:
gfortran advecMod.f -o advecMod.o && ./advecMod.o

Output file: **trajMod.out**.

6.2 Model parameters and relative dispersion metrics

To decompose the model velocities by low-passing the velocity fields and deriving the statistical parameters of the model.

In **paramPrior.h**, specify:
- Launch day, duration and particle number must be the same as in **paramMod.h**.
- The choice of the LSM to use on the shelf (*LSM*).
- The choice of the filter (*FILTER* = 'GAUSS' or 'TEMP') for the GoM interior.
- The strength of the filter for the GoM interior (*np* for the temporal average or *sigmaInt* for the Gauss-weighted average).
- If LSM=3 on the shelf, the filter is already set to be a Gaussian average. The strength can be modified by changing the value of *sigmaShelf*.

In **prior.f**, specify:
- The name/path of the filtered velocities for the GoM interior in*flvelLP*.
- The name/path of the filtered velocities for the shelf in *flvelLPs* if LSM-3 is used.
- If the filtered velocities are already computed, comment-in the instruction line "go to 200" at line 26.
- The subroutine liap: the smallest and largest spatial scales are set by the scale indices *imim* and *imax*. Here *imim*=1 corresponds to δ=R0=1. km.

Compile and run **prior.f**:
gfortran prior.f -o prior.o && ./prior.o

Output files (As of now):
autoCorrMod.dat, **sigmaTauMod.dat**.
(if LSM=3) **autocModShelf.dat**, **sigmaTauModShelf.dat**
liapMod.dat, **reldMod.dat**, **liapModShelf.dat**, **reldModShelf.dat**.

6.3 Parameterized trajectories

To advect the particles with the model and LSMs.

In **paramLSM.h**, specify:
- The number of launches (*ktm0*), the number of η_0-divisions *ne* and the phase *kphi*.
- The critical depth (*depthC*) for the transition between the two regimes.
- Launch day, duration and particle number (same as above).

(1) for the GoM interior:
- The LSM-3 statistical parameters of the model for the GoM interior: $sigu_m$, $tauu_m$, $sigv_m$, $tauv_m$, by entering the values stored in **sigmaTauMod.dat**.
- The target/'real' statistical parameters of the GoM interior: $sigu_r$, $tauu_r$, $sigv_r$, $tauv_r$ as a function of the model parameters.
(2) for the shelf:

- The choice of the LSM (*LSM*).
- If *LSM*=1: the random kick standard deviation *Lkick* and space-correlation scale *Ldecor* (for the uncorrelated case set *Ldecor*=0.001).
- If *LSM*=2: the x,y component of the turbulent velocity standard deviations *sigu, sigv* and decorrelation time scales *tauu, tauv*, as well as the space-correlation scale *Ldecor* .
- If *LSM*=3: the statistical parameters of the model for the shelf: $siguShelf_m$, $tauuShelf_m$, $sigvShelf_m$, $tauvShelf_m$, and the target parameters $siguShelf_r$, $tauuShelf_r$, $sigvShelf_r$, $tauvShelf_r$.

In **advecLSM.f**, specify:
- The name/path of the filtered velocities for the GoM interior in *flvelF*, and the number *lo* if the number of characters differs from the *flvelF* default name.
- If *LSM*=3: the name/path of the filtered velocities for the shelf in *flvelShelf* and corresponding number *los*.
- The name of the launch file in *flIni*.
- The expression of the launch-increments relating *l0* to *day0* and the loop *ktkt*.
- If *LSM*=1 on the shelf: which subroutine to use (*RwLSM1a* or *RwLSM1b*) by commenting out the other one.

Compile and run **advecLSM.f**:
gfortran advecLSM.f -o advecLSM.o && ./advecLSM.o

Output files: **traj.out.001**, **infoRun.dat**.

6.4 LSM performance tests

To compute the corrected statistical parameters and the relative dispersion metrics of the parameterized trajectories.

In **paramPost.h**, specify:
- Launch day, duration and particle number (same as above).
- The choice of the LSM (*LSM*).

In **Post.f**, specify:
- The name of the trajectory file in *fltrajR*, if it is different from **traj.out.001**.
- The name/path of the filtered velocities for the GoM interior in *flvelF*. - If *LSM*=3: the name/path of the filtered velocities for the shelf in *flvelF*.
- The lower bounds of the depth (*dpmin* and *dpmax*) for both the interior and the shelf.

Compile and run **post.f**:
gfortran post.f -o post.o && ./post.o

Output files:
autoCorrLSMInt.dat, **sigmaTauLSMInt.dat**,
liapLSMInt.dat, **reldLSMInt.dat**.
(if LSM=3:) **autocLSMShelf.dat**, **sigmaTauLSMShelf.dat**,
liapLSMShelf.dat, **reldLSMShelf.dat**.

7 Other subroutines

subroutine convLonLat(xx,yy,ktm,firstDay,depth,dpmin,dpmax,plon,plat,xxl,yyl)
Converts the grid-coordinates of *(xx, yy)* into longitudes and latitudes *(xxl, yyl)* if the positions correspond to depth in the range [*dpmin, dpmax]*.

subroutine distanceLonLat(lon1,lat1,lon2,lat2,dist)
Computes the distance *dist* (units in km) between two points on the Earth surface defined by their longitudes and latitudes *(lon1,lat1)* and *(lon2,lat2)*.

subroutine initialize(x01,y01,xprime1,yprime1,dWx1,dWy1)
Initializes the variables of the LSM-2 subroutine **EulerLSM2**.

subroutine INTERP(xp,yp,u,uu,nu)
Computes the velocity component *uu* at location *(xp,yp)* from the field *u* by a 3rd order polynomial interpolation.The subroutines used are **POLIN2(X1A,X2A,YA,M,N,X1,X2,Y,DY)** and **POLINT(XA,YA,N,X,Y,DY)** from "numerical recipes in Fortran".
The number *nu* specifies where the values of *u* are located on the C-grid.

subroutine readGridSpecs(plon,plat,qlon,qlat,ulon,ulat,vlon,vlat, pang,pscx,pscy,qscx,qscy,uscx,uscy,vscx,vscy,cori,pasp,depth)
Reads the HYCOM1/25° grid variables from the generic file *regional.grid.a*.

subroutine readTrajs(fltrajR,xx,yy,ktm,firstDay,intv)
Reads the trajectories from the file *fltrajR* and converts their x and y-components into 2D arrays *(xx, yy)* (grid-point units).

subroutine read_uv(l,flvel,u,v)
Reads the HYCOM1/25° surface velocities *(u,v)* at output iteration *l* from the file *flvel*.

subroutine read_uvF(l,flvelF,lo,u,v)
Reads the low-passed velocity fields *(u,v)* at output iteration *l* from the file *flvelF*. *lo* is an integer corresponding to the number of characters of the directory name where the fields are stored.

RK4(n,m,X0,Y0,u1,v1,u2,v2,it,iti,DT,dx,dy)
Advects a particle with a fourth-order Runge-Kutta scheme at grid-location *(X0,Y0)* with time-step *Dt* , and at the interpolated time indice *it* from the consecutive velocity outputs *(u1,v1)* and *(u2,v2)*.

subroutine timeLinear(n,m,x0,y0,u1,v1,u2,v2,dx,dy,it,iti,uhor,vhor)
Interpolates linearly in time two consecutive velocity output fields *(u1,v1)* and *(u2,v2)* from HYCOM1/25° near the particle location *(x0,y0)* on the horizontal grid.

subroutine zeroTheLand(u,v))
Removes the land mask and set to zero the velocity components above land.

8 List of variables and parameters

MODEL (HYCOM1/25°):
n,m : domain dimensions in the zonal and meridional axes, respectively.
dx(m),dy(m) : grid spacing.
tHycom : time-interval between consecutive outputs (units in seconds).
u(n,m),v(n,m) : surface velocity components of the model (units= m/s).
umean(n,m),vmean(n,m) : low-passed velocities for the LSM-3 of the GoM interior.
umeans(n,m),vmeans(n,m) : low-passed velocities for the LSM-3 of the shelf.

TIME INTEGRATION:
Dt : integration time-step (units = seconds).
iti : total time increments of the HYCOM interpolated velocities.
ttime : time (unit = days).
elaps : elapsed time since the particle launch (unit = days).
day0 : day of launch or start of a time-loop.
lmt : total integration period (unit = model time-intervals).
l0 : launch day for each time-loop.

PARTICLES:
npar : number of particles released.
x0i(npar),y0i(npar) : particle positions at launch (units = grid points).
x0(npar),y0(npar) : particle positions of the model (HYCOM) trajectories.
xc(npar),yc(npar) : particle positions of the parameterized trajectories.

SHELF/INTERIOR TRANSITION:
LSM : integer specifying the Lagrangian Stochastic model used on the continental shelf.

depthC : critical depth separating the treatment of the shelf and GoM interior (units in meters). It is set here at 70 m to allow mesoscale intrusions on the shelf.

AUTOCORRELATION:
udet(npar,ktmax),vdet(npar,ktmax) : detrended Lagrangian velocities.
cuum(ktmax),cvvm(ktmax) : autocovariance functions of the detrended Lagrangian velocities.

GOM INTERIOR:

LSM-3:
sigu_m,tauu_m,sigv_m,tauv_m : velocity fluctuation stds and decorrelation time-scales of the model trajectories in both zonal and meridional directions.
sigu_r,tauu_r,sigv_r,tauv_r : same parameters as above for the assumed "real"/target trajectories.
ax,bx,cx : zonal coefficients for the equation governing the evolution of the eta-vector.
ay,by,cy : corresponding meridional coefficients.
sex,sey : expected standard deviations of the eta-vector components.
ktm0 : number of launches (in time).
ne : number of initial eta_0s.
kphi : phase between etax0 and etay0.
etax0(ktm0),etay0(ktm0) : zonal and meridional components of the initial eta-vectors (initially independent of the particle).
eex(npar),eey(npar) : zonal and meridional components of the eta-vectors for each particle.

SHELF:

LSM-1 (RANDOM WALK):
Lkick : standard deviation of the random kick during the integration time-step Dt (units = km).
Ldecor : decorrelation space scale (also used in the LSM-2) for spatial correlation (units = km).
dWx,dWy : random components for the zonal and meridional directions from the normal distribution with a with of unity and zero-mean (no dimension).

LSM-2 (RANDOM FLIGHT):
sigu,sigv : standard deviation of the velocity fluctuations in the zonal and meridional direc-

tions, respectively (units = cm/s).
tauu,tauv : decorrelation time scales in the zonal and meridional directions (units = days).

LSM-3:
siguShelf_m,tauuShelf_m,sigvShelf_m,tauvShelf_m : velocity fluctuation stds and decorrelation time-scales of the model trajectories on the shelf in both zonal and meridional directions.
siguShelf_r,tauuShelf_r,sigvShelf_r,tauvShelf_r : same parameters as above for the "real"/target shelf trajectories.
axs,bxs,cxs : zonal coefficients for the equation governing the evolution of the eta-vector on the shelf.
ays,bys,cys : corresponding meridional coefficients.

RELATIVE DISPERSION STATISTICS:
nps : number of satellite particles per central particle. For triplet configurations, nps=2.
dispM(ktmax) : time-dependent relative dispersion $D^2(t)$ averaged over all the particle trajectories.
im2 : total number of alpha-indices.
liapS(im2) : scale-dependent FSLE ($\lambda(\delta)$) as a function of the scale indice.
R0 : initial relative distance (FSLE).
Rimi,R i: scales R_{i-1} and R_i.
alpha : ratio of R_i over R_{i-1}.
delt(im2) : averaged time ($<\tau>$) taken by all particles pairs to separate from R_{i-1} to R_i.
delt2(im2) : averaged square-time ($<\tau^2>$).
icount(im2) : number of pairs available to compute $\lambda(\delta)$ as a function of the spatial scale-indice.